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Heat Regulation in Human Dermal Layers with Atmosphere Based Metabolic Activity

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ABSTRACT: In this paper we deal with the study of one dimensional temperature distribution in human skin and subcutaneous tissue. Skin is divided into three major layers namely epidermis, dermis and subcutaneous tissue. According to biological structure of the region, the rate of blood mass flow and the tissue thermal conductivity are assumed to be constants in epidermis and subcutaneous tissue but variables in dermis. The rate of metabolic heat generation in active regions is dependent on the tissue and the atmospheric temperature. Mathematical model involving partial differential equation is solved with the help of variational finite element method. Theoretical results are discussed numerically and graphically.

Keywords: Human Skin, Temperature Distribution, Variational Approach, Finite Element Method.

Mathematics Subject Classification (2010): 92C35

I. INTRODUCTION

We investigate the temperature distribution in dermal parts (skin and subcutaneous tissue) of human body one dimensional heat equation. The temperature regulation in outer parts of human body is highly impacted by the atmospheric conditions. The generated metabolic heat in cell regions is absorbed or released as per the biological requirements so that no access energy harms the internal parts. At the same time, low tissue temperature causes heat release. The human body maintains its fixed core temperature under different atmospheric conditions. Parameters like rate of blood flow, rate of metabolic heat generation and thermal conductivity play important role to maintain it. The human dermal region is divided in three layers, epidermis, dermis and subcutaneous tissue. Epidermis contains dead cells, hence there is no blood flow and no metabolic activity is available near the interface of epidermis and dermis. This activity is very small and it can be neglected. The next layer dermis contains various constituents such as blood capillaries, blood cells, lymphatics, nerves, sweat glands etc. The next layer subcutaneous tissue is fully saturated with active cells and blood vessels. Population density of cells increasing gradually and becomes almost uniform in the subcutaneous tissue.

In epidermis and subcutaneous tissue parameters are taken as constant but in dermis these are in composite form with dependence on position and tissue temperature.

Based on the initial model of heat flow given by W. Perl [1] the governing partial differential equation is given below:

$$\rho \bar{c} \frac{\partial u}{\partial t} = div\left(K \, grad \, u\right) + m_b c_b \left(u_b - u\right) + S \tag{1}$$

Where ρ , $\bar{c}$, K and S are respectively tissue density, specific heat of tissue, thermal conductivity of tissue and rate of metabolic heat generation. m_b and c_b are respectively, mass blood flow rate and specific heat of blood. Heat regulation in outer parts of human body has been studies by several investigators from time to time. W. Perl [1], Chao *et.al* [2], Cooper and Trezek [3], Gray and Steketee *et.al* [4,5] used this equation under simplified situations. To study problems of heat flow Saxena [6,7], Saxena and Arya [8,9], Saxena and Bindra [10,11], used this model. Gurung, Saxena and Adhikari [12,13], Saxena and Khanday [14], Sharma and Tripathi [15]studied cold related problems of human skin.Vaile *et.al* [16] studied cold water effect on human skin and limbs, Goodall and Howatson [17] find effect of cold water in skin and muscle damage. In this paper we have improved the earlier work by taking better conditions.

Here we modeled human skin divided in three layers and with idealized scheme of division is given below:

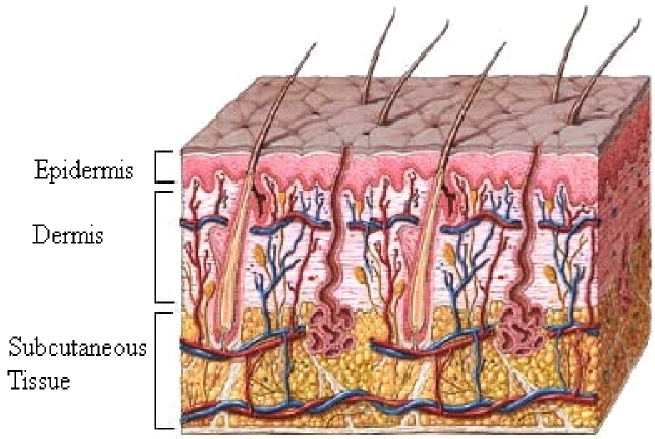

Fig. 1 (a): Biological Structure of Human Skin.

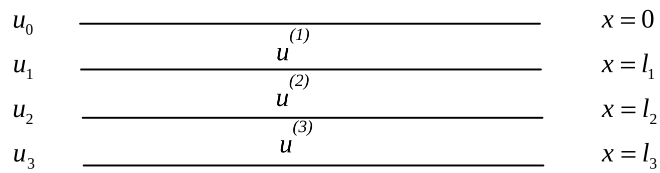

Fig. 1 (b): Representative form of Skin Layers.

Here u_0 is temperature of the outer layer of the skin u_1, u_2, u_3 are nodal temperature of skin layers and l_1, l_2, l_3 are layer distances from the outer surface.

II. MATHEMATICAL FORMULATION

Equation (1) in one dimensional steady state form can be written as:

$$\frac{d}{dx}\left(K\frac{du}{dx}\right)+m_b c_b\left(u_b-u\right)+S\left(u-u_a\right)=0 \tag{2}$$

The last term on the left indicates generation or absorption of heat in active tissue as per requirement. The region is divided in three layers $u_b$and u_a are body core temperature and atmospheric temperature respectively.
Comparing equation (2) to Euler-Lagrange's equation (Myers [18]).

$$\frac{d}{dx}\left(\frac{\partial f}{\partial u'}\right)-\frac{\partial f}{\partial u}=0,$$

Where $u'=\frac{du}{dx}$

This is equivalent to the optimum value of the variational form.

$$I_{opt}=\int_0^{l_i} f\,dx$$

Here I_{opt} indicates optimum value of the integral.
Thus, the variational form of equation (2) for i^{th} layer is given as:

$$I_i=\frac{1}{2}\int_{l_{i-1}}^{l_i}\left[K_i\left(\frac{du^{(i)}}{dx}\right)^2+\left(M_i-S_i\right)\left(u^{(i)}\right)^2+2\left(S_i u_a-M_i u_b\right)u^{(i)}\right]dx \tag{3}$$

As the variations are not large and the thicknesses are small, we take following linear shape functions for each region:

$$u^{(i)} = A_i + B_i\, x \text{ for } i = 1,2,3.$$

where

$$A_i = \frac{u_{i-1} l_i - u_i l_{i-1}}{l_i - l_{i-1}}\ ,\ B_i = \frac{u_i - u_{i-1}}{l_i - l_{i-1}}$$

Assumptions: Considering the layer wise activities, make following assumptions.

Layer -I

As the layers contains mostly dead tissue with no active cells and micro-circulation, thus
K_1 = constant, $M_1 = S_1 = 0$ in the first layer.

Taking the first layer; we get

$$I_1 = \frac{1}{2} K_1\, B_1^2\, l_1 = \alpha_1 \left(u_1 - u_0\right)^2 \tag{4}$$

Where

$$\alpha_1 = \frac{1K_1}{2l_1}$$

Layer -II

Inside the second layer (i = 2) all the parameters are behaving actively and depend on positions. Therefore K_2 and M_2 are functions of x. It is good to assume

$$K_2 = \frac{\left(x - l_1\right) K_3}{\left(l_2 - l_1\right)} + \frac{\left(l_2 - x\right) K_1}{\left(l_2 - l_1\right)}\ ,\ \ M_2 = \frac{\left(x - l_1\right) m}{\left(l_2 - l_1\right)}$$

There is no significant change in S_2.

Accordingly on integration we obtain

$$I_2 = \beta_1 (u_1^2 + u_2^2 - 2u_1 u_2) + \beta_2 (u_1^2 l_2^2 + u_2^2 l_1^2 - 2u_1 u_2 l_1 l_2) + \beta_3 (u_1 u_2 l_2 - u_2^2 l_2 - u_1 u_2 l_1)$$
$$+ \beta_4 (u_1^2 + u_2^2 + u_1 u_2) + \beta_5 u_2 + \beta_6 u_1$$

or

$$I_2 = \beta_7 u_1^2 + \beta_8 u_2^2 + \beta_9 u_1 u_2 + \beta_5 u_2 + \beta_6 u_1 \quad \ldots(5)$$

Where β's are defined in the Appendix.

Layer -III

In this layer ($i = 3$) there is no much effect of the position due to biological homogeneity. Thus,

$$I_3 = \frac{1}{3} K_3 \frac{(u^3 - u^2)^2}{(l_3 - l_1)} + \frac{1}{2}(M_3 - S_3)\frac{(u^3{}_2 + u_2{}^2 + u_3 u_2)}{3}(l_3 - l_2) + (S_3 u_a - M_3 u_b)\frac{(u_3 + u_2)(l_2 - l_1)}{2}$$

or

$$I_3 \quad = \gamma_1 u_2{}^2 + \gamma_1 u_3{}^2 + \gamma_2 u_2 u_3 + \gamma_3 u_3 + \gamma_3 u_2 \quad \ldots(6)$$

Where γ's are defined in the Appendix.

The total variational integral I can be represented as

$$I = \sum_{i=1}^{3} I_i$$

This gives

$$I=\alpha_1(u_1-u_0)^2+\beta_7u_1^2+\beta_8u_2^2+\beta_9u_1u_2+\beta_5u_2+\beta_6u_1+\gamma_1u_2^2+\gamma_1u_3^2+\gamma_2u_2u_3+\gamma_3u_2+\gamma_3u_3$$

or

$$I=\alpha_1u_0^2+\lambda_1u_1^2+\lambda_2u_2^2+\gamma_1u_3^2-2\alpha_1u_0u_1+\beta_9u_1u_2+\gamma_2u_2u_3+\beta_6u_1+\lambda_3u_2+\gamma_3u_3 \quad \ldots(7)$$

Where λ's are defined in the Appendix.

Optimizing I with respect unknown nodal temperature u_1 and u_2. We arrive at the following equations

$$A\,U=B \quad \ldots(8)$$

Where $A=\begin{bmatrix}2\lambda_1 & \beta_9\\ \beta_9 & 2\lambda_2\end{bmatrix}$, $U=\begin{bmatrix}u_1\\ u_2\end{bmatrix}$ and $B=\begin{bmatrix}\lambda_4\\ \lambda_5\end{bmatrix}$.

III. NUMERICAL COMPUTATION

We solve the system of equation (8) for the following values of constants and parameters (Bindra[10,11]).

$K_1=0.030\,Cal/cm-\min^o C$, $K_3=0.060\,Cal/cm-\min^o C$, $\rho=1.05\ gm/cm^3$, $\bar{c}=0.83\ cal/gm$.

We can assign different values to the constants l_1, l_2 and l_3 depending on the sample of the skin and subcutaneous tissue under study. The set of values considered here are as follows:

Table 1

Set	l_1(cm)	l_2(cm)	l_3(cm)
I	0.10	0.35	0.50
II	0.10	0.40	0.90

The numerical calculations have been made for the following three cases of atmospheric temperature together with the respective values of *S* and *M*.

Table 2

Atmospheric Temperature u_a(°C)	$M=M_{max}=(m_bc_b)_{max}$ (cal/cm^3-min° C)	$S=S_{max}$ (cal/cm^3-min°C)
15	0.003	0.0357
23	0.018	0.018
33	0.0315	0.018

The values in Table 1 are only some trivial samples which can be extended to more data for specific for parts and individuals. The values in Table 2 are some standard considerations.

Set-I

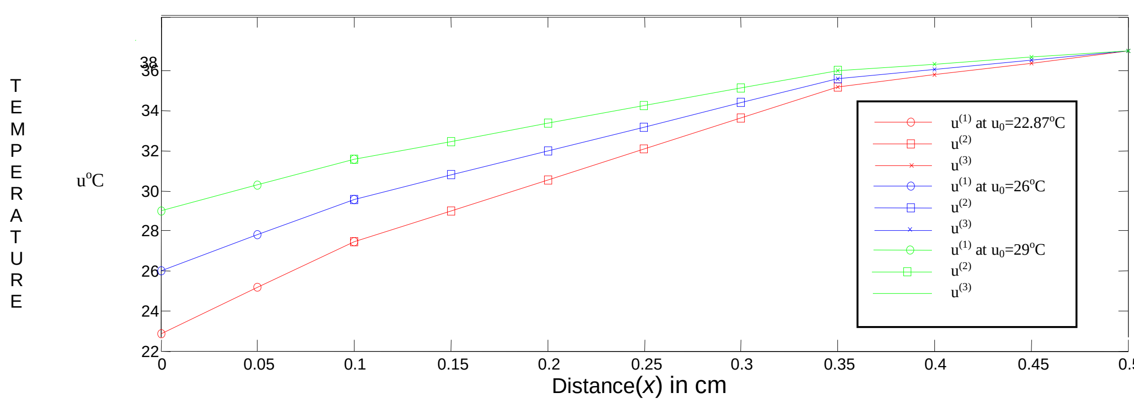

Fig. 2. Graph between distance and temperature at u_a = 15^{0}C, u_0 = 22.87^{0}C, u_0 = 26^{0}C, u_0 = 29^{0}C.

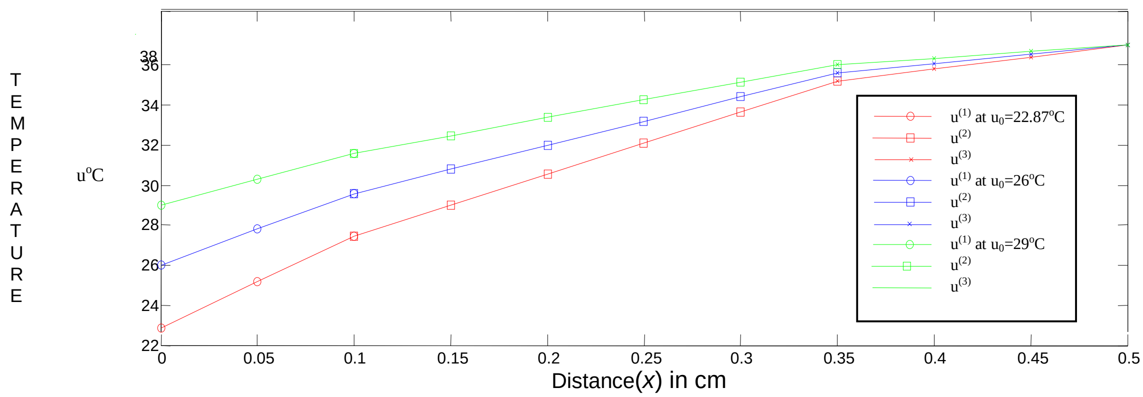

Fig. 3. Graph between distance and temperature at $u_a = 23^0$C, $u_0 = 22.87^0$C, $u_0 = 26^0$C, $u_0 = 29^0$C.

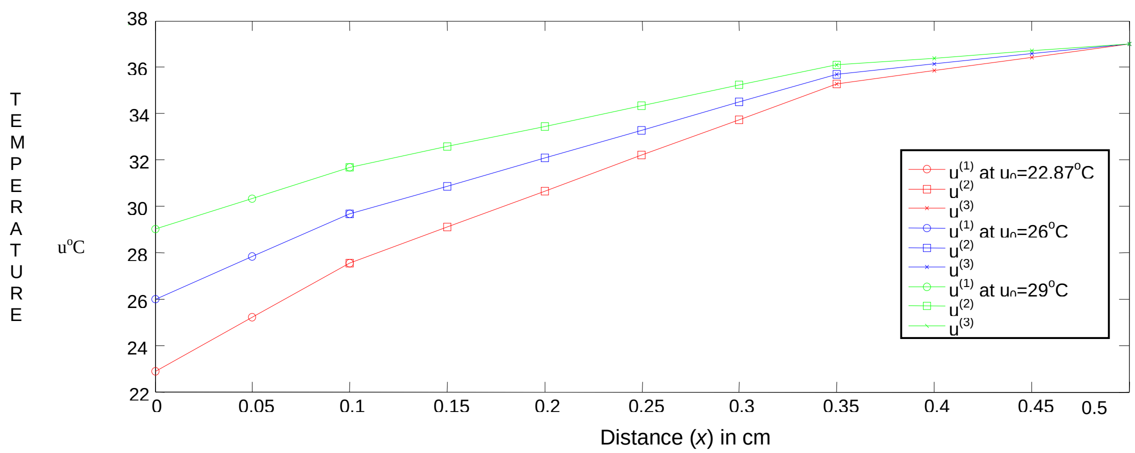

Fig. 4. Graph between distance and temperature at $u_a = 33^0$C, $u_0 = 22.87^0$C, $u_0 = 26^0$C, $u_0 = 29^0$C.

Set-II

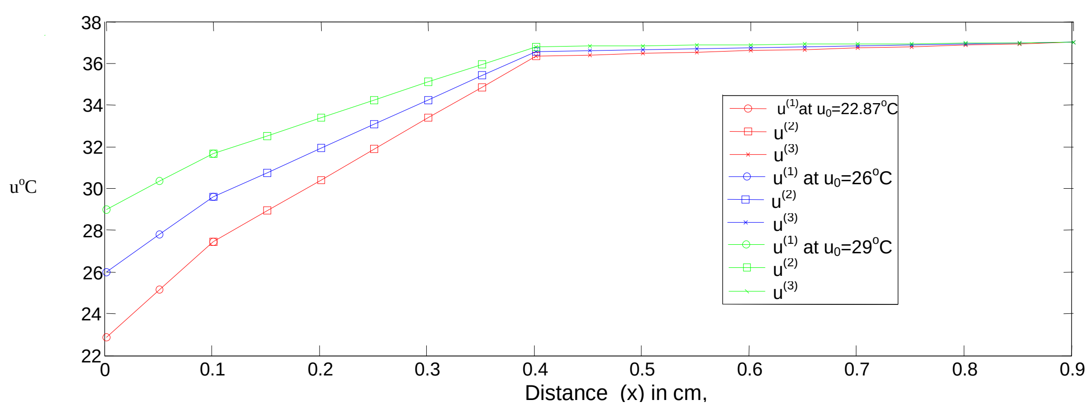

Fig. 5. Graph between distance and temperature at $u_a = 15^0$C, $u_0 = 22.87^0$C, $u_0 = 26^0$C, $u_0 = 29^0$C.

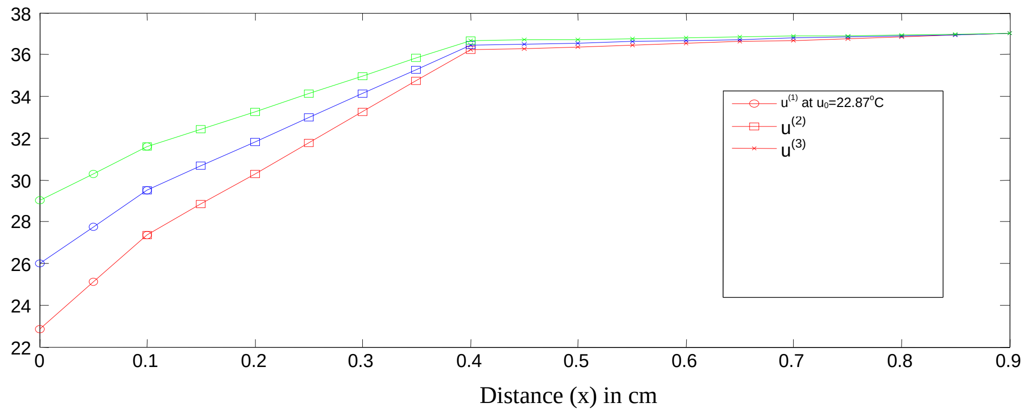

Fig. 6. Graph between distance and temperature at $u_a = 23^0$C, $u_0 = 22.87^0$C, $u_0 = 26^0$C, $u_0 = 29^0$C.

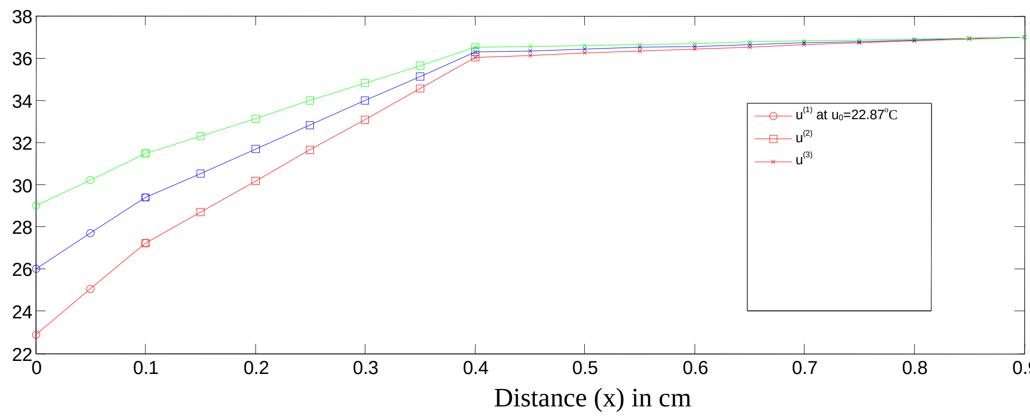

Fig. 7. Graph between distance and temperature at $u_a = 33^0$C, $u_0 = 22.87^0$C, $u_0 = 26^0$C, $u_0 = 29^0$C.

IV. CONCLUSION

There are six graphs (Figs. 2-7) for two sets of thickness values of dermal layers. Each graph considers different atmospheric temperature. The natures of graphs clearly indicate the variation of temperatures in respective cases. The temperature variation in first case is fast for first set while it is slow in the internal parts of the region. The distances in the profiles are also different for different cases. Above calculations are carried out using MATLAB and is only an example. A variety of other physiological cases can also be carried out. The model under study depicts only equilibrium case where no change occurs with respect to time. However, there are situations where time dependence in inevitable.

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Appendix

$$\beta_1 = \frac{(K_3+K_1)}{4(l_2-l_1)} + \frac{1}{2}\alpha_3(l_2^{\,2}+l_1^{\,2})$$

$$\beta_2 = \frac{m(l_2+l_1)}{4(l_2-l_1)^2}$$

$$\beta_3 = \frac{m}{3}(l_2^{\,2}+l_1^{\,2}+l_1l_2)/(l_2-l_1)^2$$

$$\beta_4 = -\frac{1}{6}\left(\frac{ml_1}{l_2-l_1}+S_2\right)(l_2-l_1)$$

$$\beta_5 = \left\{S_2u_a + \frac{ml_1u_b}{(l_2-l_1)}\right\}\frac{(l_2-l_1)}{2} - \frac{mu_b}{3}(l_2^{\,2}+l_1^{\,2}+l_1l_2)/(l_2-l_1) + \frac{mu_b}{2}(l_2+l_1)l_1/(l_2-l_1)$$

$$\beta_6 = \left\{S_2u_a + \frac{ml_1u_b}{(l_2-l_1)}\right\}\frac{(l_2-l_1)}{2} + \frac{mu_b}{3}(l_2^{\,2}+l_1^{\,2}+l_1l_2)/(l_2-l_1) - \frac{mu_b}{2}(l_2+l_1)l_2/(l_2-l_1)$$

$$\beta_7 = \beta_1 + \beta_2l_2^{\,2} - \beta_3l_2 + \beta_4$$

$$\beta_8 = \beta_1 + \beta_2l_1^{\,2} - \beta_3l_1 + \beta_4$$

$$\beta_9 = -2\beta_1 - 2\beta_2l_1l_2 + \beta_3(l_1+l_2) + \beta_4$$

$$\gamma_1 = \frac{K_3}{2(l_3-l_2)} + \frac{1}{6}(M_3-S_3)(l_3-l_2)$$

$$\gamma_2 = \frac{1}{6}(M_3-S_3)(l_3-l_2) - 2\frac{K_3}{2(l_3-l_2)}$$

$$\gamma_3 = \frac{(S_3 u_a - M_3 u_b)(l_2 - l_1)}{2}$$

$$\lambda_1 = \alpha_1 + \beta_7$$

$$\lambda_2 = \lambda_8 + \gamma_1$$

$$\lambda_3 = \beta_5 + \gamma_3$$

$$\lambda_4 = 2\alpha_1 u_0 - \beta_6$$

$$\lambda_5 = -\gamma_2 u_3 - \lambda_3$$